DIRECTION FINDING USING SPECTRAL ESTIMATION WITH ARBITRARY ANTENNA ARRAYS

Tuan Do-Hong, Walter Fisch, Peter Russer

Lehrstuhl für Hochfrequenztechnik,
Technische Universität München.
Arcisstrasse 21
80333 München, Germany.

Abstract **This paper presents direction-finding techniques for smart antennas based on conformal and irregular antenna arrays. Methods of spectral estimation are considered to estimate directions-of-arrival (azimuth and elevation) using two-dimensional arrays and three-dimensional arrays (conformal arrays). The effects of arbitrary arrays on the performance and on the resolution of direction-finding algorithms are illustrated by simulation results.**

I. INTRODUCTION

Future millimeter wave mobile communications systems will also use smart antennas in mobile stations. For this application conformal smart antennas will be of high interest. This requires direction-finding methods suitable for conformal and irregular antenna arrays. This paper describes the extension of methods for direction finding suitable for conformal and irregular antenna arrays. Direction-of-arrival estimation techniques using spectral estimation methods have been studied over the past decade with attention given mainly to linear antenna arrays (regular one-dimensional arrays) and rectangular antenna arrays (regular two-dimensional arrays). In this paper we examine the effects of arbitrary two-dimensional (2-D) and three-dimensional (3-D) arrays on the performance and resolution of direction-finding algorithms based on spectral estimation methods. Three algorithms used in this paper include of Blackman-Tukey (BT), Minimum Variance Spectral Estimator (MVSE) and Multiple Signal Classification (MUSIC). The details of the algorithms are introduced in [1].

The next section describes a signal and noise models which are used normally for direction finding; then, in section III and IV, the array steering matrices for arbitrary 2-D and 3-D arrays are determined; in section V, spectral estimation algorithms are introduced in summary; finally, in section VI the simulation results will be shown.

II. MODELS OF SIGNAL AND NOISE

Antenna arrays are characterized by their spatial geometry and by the directional patterns of the antenna elements. In the paper, it is assumed that the antennas have uniform directional patterns and are placed arbitrarily on the array. Each element is assumed acting independently of all the others, so the mutual coupling between two elements can be approximated as zero. The signals impinging on the arrays are characterized by their bandwidth and by their mutual correlation. The impinging signals can be uncorrelated when they are radiated from independent sources, or can be partially or fully correlated as in multipath propagation. For the simulation, we consider the case of narrowband signals, which are transmitted from independent sources.

Consider an array composed of M antennas with arbitrary locations. Assume that p narrowband sources, in far field of array, centered around a known frequency, say ω_0, impinging on the array from distinct locations $\theta_1,...,\theta_p$. In this case the parameter that characterizes the source locations is the direction-of-arrival (DOA).

Using complex envelope representation, the $(M\times1)$-signal vector received by the array can be expressed by

$$\mathbf{x}(t) = \sum_{k=1}^{p} \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t) \qquad (1)$$

where $\mathbf{a}(\theta)$ is the $(M\times1)$-steering vector of the array toward direction θ

$$\mathbf{a}(\theta) = \left[\mathrm{a}_1(\theta)\, e^{-j\omega_0\tau_1(\theta)}, ..., \mathrm{a}_M(\theta)\, e^{-j\omega_0\tau_M(\theta)} \right]^T \qquad (2)$$

$\mathrm{a}_k(\theta)$ denotes the amplitude response of the k-th antenna element toward direction θ; $\tau_k(\theta)$ denotes propagation delay between the reference point and the k-th element; $\mathrm{s}_k(t)$ denotes the signal of the k-th source as received at the reference point, and $\mathbf{n}(t)$ denotes the $(M\times1)$-vector of noise at the antenna elements. In matrix notation, it is

$$\mathbf{x}(t) = \mathbf{A}(\Theta)\mathbf{s}(t) + \mathbf{n}(t) \qquad (3)$$

where $\mathbf{A}(\Theta)$ is the $(M\times p)$-matrix of the steering vectors. The number M of antenna elements is assumed to satisfy the condition:

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$M \geq p+D$, where D = 1, 2, 3 for 1-D, 2-D, 3-D estimations, respectively [4].

Therefore the matrix $\mathbf{A}(\Theta)$ has full rank.

$$\mathbf{A}(\Theta) = [\mathbf{a}(\theta_1),...,\mathbf{a}(\theta_p)] \quad (4)$$

Suppose that the received vector $\mathbf{x}(t)$ is sampled N times (N snapshots), at $t_1,...,t_N$. From (1), the sampled data can be expressed as

$$\mathbf{X} = \mathbf{A}(\Theta)\mathbf{S} + \mathbf{N} \quad (5)$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $(M\times N)$-matrices

$$\mathbf{X} = [\mathbf{x}(t_1),...,\mathbf{x}(t_N)] \qquad \mathbf{N} = [\mathbf{n}(t_1),...,\mathbf{n}(t_N)]$$

and $\mathbf{S}$ is the $(p\times N)$-signal matrix

$$\mathbf{S} = [\mathbf{s}(t_1),...,\mathbf{s}(t_N)]$$

Then, an estimation of the covariance matrix $\mathbf{R}_{xx}$ is given by

$$\hat{\mathbf{R}}_{xx} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(t_n)\mathbf{x}^H(t_n) = \frac{1}{N}\mathbf{X}\mathbf{X}^H \quad (6)$$

The model in (3) needs to be complemented with some additional assumptions:

- The signal $\mathbf{s}(t_i)$ are referred to as Deterministic Signals (DS) [2], which are fixed for all ensemble realization.
- The samples $\mathbf{n}(t_i)$ are statistical Gaussian random vectors with zero mean and covariance matrix $\sigma^2\mathbf{I}$.

III. Arbitrary 2-D Arrays

Consider a two-dimensional (2-D) uniform antenna array of M elements lying on the x-y plane as depicted in Fig. 1.The impinging signals on the array are p narrowband wavefronts with wavelength λ, azimuth ϕ_i and elevation θ_i, $1 \leq i \leq p$. The Fig. 1 also defines the range of azimuth ($-180° \leq \phi_i \leq 180°$) and elevation ($0° \leq \theta_i \leq 90°$). We assume that the estimated positions are on the unit sphere. The direction cosines x_i and y_i are the rectangular coordinates of the projection of the corresponding point on the unit sphere on the equatorial plane.

Let

$$\begin{cases} x_i = \cos\phi_i \sin\theta_i \\ y_i = \sin\phi_i \sin\theta_i \end{cases} \quad (7)$$

where $1 \leq i \leq p$, denote the direction cosines of the i-th source relative to the x- and y- axes, respectively, as in Fig. 1.

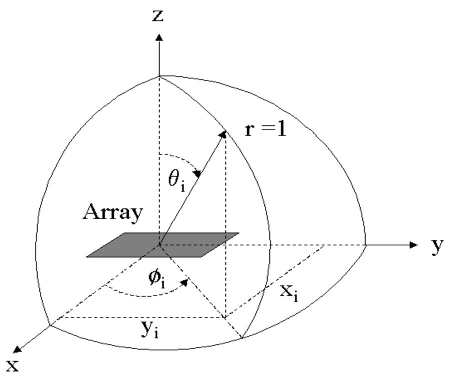


Fig. 1. Two-dimensional array of M antenna elements and the definitions of azimuth and elevation of DOA.

Now considering the 2-D uniform array of M elements placed arbitrarily on the array as depicted in Fig. 2. Let $\mathbf{d}_j = [d_{xj}, d_{yj}]^T$ ($1 \leq j \leq M$) denote the location of the j-th element in antenna plane. Define the first element as the reference element, so $d_{x1} = d_{y1} = 0$.

In the case of 2-D estimation, the spatial frequencies of an arbitrary array are defined as
where x_i, y_i are defined as in (7)

$$\begin{cases} \mu_i = \dfrac{2\pi}{\lambda} x_i \\ \upsilon_i = \dfrac{2\pi}{\lambda} y_i \end{cases} \quad (8)$$

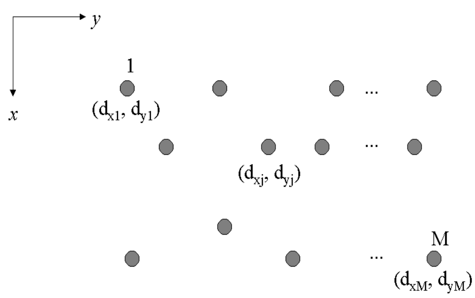


Fig. 2. Arbitrary 2-D array of M antenna elements

Let $\mathbf{\Omega}_i = [\mu_i, \upsilon_i]^T$ denotes the spatial frequency in vector form. Therefore, we can determine the complex 1-D steering vector of size M with the first element as reference:

$$\mathbf{a}(\mu_i, \upsilon_i) = \left[1, e^{j(d_{x2}\mu_i + d_{y2}\upsilon_i)},..., e^{j(d_{xM}\mu_i + d_{yM}\upsilon_i)}\right]^T$$

$$= \left[1, e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_i}, \ldots, e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_i} \right]^T \qquad (9)$$

Hence, the array steering matrix has the following form as

$$\mathbf{A}(\mu,\upsilon) = \begin{bmatrix} 1 & 1 & .. & .. & 1 \\ e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_1} & e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_2} & .. & .. & e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_p} \\ e^{j\,\mathbf{d}_3^T \mathbf{\Omega}_1} & e^{j\,\mathbf{d}_3^T \mathbf{\Omega}_2} & .. & .. & e^{j\,\mathbf{d}_3^T \mathbf{\Omega}_p} \\ . & . & . & . & . \\ . & . & . & . & . \\ e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_1} & e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_2} & .. & .. & e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_p} \end{bmatrix} \qquad (10)$$

IV. Arbitrary 3-D Array

In this section we examine a three-dimensional antenna array of M elements, which are placed arbitrarily in x-y-z space. Let $\mathbf{d}_j = [d_{xj}, d_{yj}, d_{zj}]^T$ $(1 \le j \le M)$ denotes the location of the j-th element in antenna space.

The impinging wavefronts on the 3-D array are caused by p narrowband signals with the wavelength λ, azimuth ϕ_i $(-180° \le \phi_i \le 180°)$, elevation θ_i $(0° \le \theta_i \le 90°)$ and the range r_i, $1 \le i \le p$. For 2-D estimation of azimuth and elevation, it is assumed that r_i=1.

Similar to 2-D arrays, let us define the spatial frequencies as

$$\begin{cases} \mu_i = \dfrac{2\pi}{\lambda} x_i \\ \upsilon_i = \dfrac{2\pi}{\lambda} y_i \\ \eta_i = \dfrac{2\pi}{\lambda} z_i \end{cases} \qquad (11)$$

where

$$\begin{cases} x_i = \cos\phi_i \sin\theta_i \\ y_i = \sin\phi_i \sin\theta_i \\ z_i = \cos\theta_i \end{cases}$$

Let $\mathbf{\Omega}_i = [\mu_i, \upsilon_i, \eta_i]^T$ denotes the spatial frequency vector, so the array steering vector of 3-D array is expressed as

$$\mathbf{a}(\mu_i, \upsilon_i, \eta_i) = \left[e^{j\,\mathbf{d}_1^T \mathbf{\Omega}_i}, e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_i}, \ldots, e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_i} \right]^T \qquad (12)$$

Define the first element as the reference, the array steering matrix of p DOAs for 3-D arrays is given by

$$\mathbf{A}(\mu,\upsilon,\eta) = \begin{bmatrix} 1 & 1 & .. & .. & 1 \\ e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_1} & e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_2} & .. & .. & e^{j\,\mathbf{d}_2^T \mathbf{\Omega}_p} \\ e^{j\,\mathbf{d}_3^T \mathbf{\Omega}_1} & e^{j\,\mathbf{d}_3^T \mathbf{\Omega}_2} & .. & .. & e^{j\,\mathbf{d}_3^T \mathbf{\Omega}_p} \\ : & : & : & : & : \\ e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_1} & e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_2} & .. & .. & e^{j\,\mathbf{d}_M^T \mathbf{\Omega}_p} \end{bmatrix} \qquad (13)$$

V. Spectral Estimation

Three spectral estimation methods considered in this paper include of the Blackman-Tukey method, which is a classical method based on Fourier transform, the MVSE and MUSIC algorithms, which are high-resolution methods. In this section, we introduce the final results obtained with these methods. A detail of their description can be found in [1], [2]. By applying the principal component method [1], the spectral estimation algorithms as BT, MVSE, MUSIC can be expressed as

$$\hat{P}_{BT}(\mu,\upsilon) = \frac{1}{M} \sum_{i=1}^{p} \lambda_i \, | \mathbf{a}^H \mathbf{v}_i |^2 \qquad (14)$$

$$\hat{P}_{MVSE}(\mu,\upsilon) = \frac{1}{\sum_{i=1}^{p} \frac{1}{\lambda_i} | \mathbf{a}^H \mathbf{v}_i |^2} \qquad (15)$$

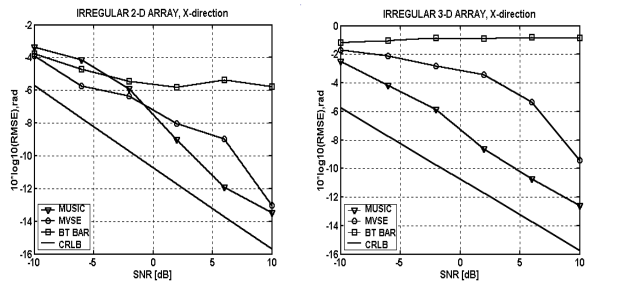
$$\hat{P}_{MUSIC}(\mu,\upsilon) = \frac{1}{\sum_{i=p+1}^{M} | \mathbf{a}^H \mathbf{v}_i |^2} \qquad (16)$$

where the steering vector $\mathbf{a}$ is determined as in section III and IV, and the vectors $\mathbf{v}_i$ $(1 \le i \le p)$ are the principal eigenvectors corresponding to the eigenvalues λ_i of estimated covariance matrix in equation (6).

VI. Simulation Results

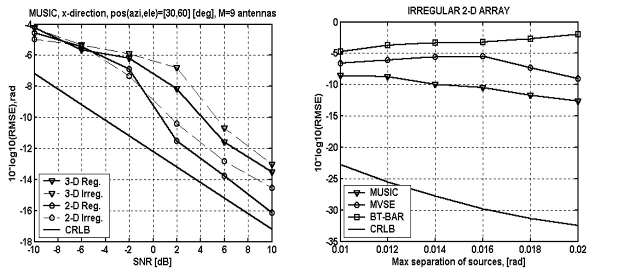
The performance is determined using the RMSE (Root Mean Square Error). To assess performance of algorithms with different antenna arrays, the Cramér-Rao Lower Bound (CRLB) introduced in [2]-[4] is used. We compare

2-D arrays and 3-D arrays in 2 cases: RMSE vs. SNR (Fig. 3a, 3b, 4a) and RMSE vs. $\delta\omega$ (Fig. 4b, 5a, 5b), here SNR is the signal to noise ratio and $\delta\omega$ is the maximum separation of spatial frequencies of the signal sources.



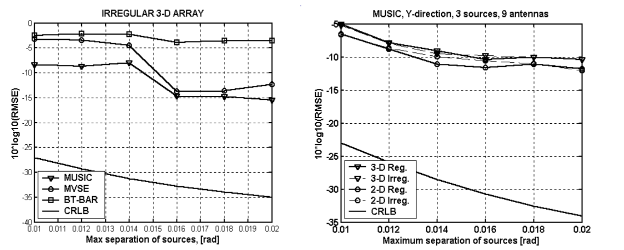
(a) (b)

Fig. 3 . RMSE of three spectral estimation algorithms using irregular array, N=100 snapshots, p=3 DOAs, azimuth of 10°, elevation of 40°. (a) 2-D array, (b) 3-D array.



(a) (b)

Fig. 4 . (a) RMSE of MUSIC vs. SNR for regular 2-D, 3-D arrays and irregular 2-D, 3-D arrays. (b) RMSE of three spectral estimations vs. $\delta\omega$ using irregular 2-D array, SNR=5dB. N=100 snapshots, p=3 DOAs, azimuth of 30°, elevation of 60°.



(a) (b)

Fig . 5 . (a) RMSE of three spectral estimation algorithms vs. $\delta\omega$ using irregular 3-D array, SNR=5dB. (b) RMSE of MUSIC vs. $\delta\omega$ for regular 2-D, 3-D arrays and irregular 2-D, 3-D arrays.

We use the regular 2-D antenna array of M=9 elements which are placed in x-y plane with distance between adjacent elements of $\lambda/2$. For regular 3-D array, the elements are placed regularly on the parabolic surface of $d_{zj}/\lambda = 0.5\,[(d_{xj}/\lambda)^2+(d_{yj}/\lambda)^2]$, $(1 \le j \le M)$ where (d_{xj}, d_{yj}) are x-y coordinates of j-th element of the array. The irregular 2-D antenna array has M=9 elements placed irregularly in x-y plane as

$\{(d_{xj}/\lambda, d_{yj}/\lambda)\}_{j=1\dots M}$ = {(0.0, 0.0); (0.45, 0.05); (0.9, 0.0); (0.1, 0.55); (0.45, 0.45); (0.95, 0.58); (0.0, 0.9); (0.5,1.0); (1.0,0.85)}.

For irregular 3-D array, the elements are also positioned on the parabolic surface as the regular 3-D array with normalized coordinates d_{xj}/λ, d_{yj}/λ are similar to irregular 2-D array.

VII. Conclusions

For future millimeter wave applications, in this paper the performance of direction-finding techniques for conformal and irregular antenna arrays are compared. By determining the array steering matrices for arbitrary 2-D and 3-D arrays and by calculating the estimated covariance matrices, the power spectral density (PSD) can be estimated with BT, MVSE or MUSIC. Then, by searching on PSD to find the peaks of spectrum, the DOAs of impinging signals on array are determined. The performance of algorithms and the resolution of signal sources with different structures of arrays are assessed using RMSE and Cramér-Rao Lower Bound (CRLB).

The simulation results show that for both irregular 2-D arrays and irregular 3-D arrays, the MUSIC algorithm is the best one, giving the performance nearly to the CRLB (Fig. 3a, and 3b). The Blackman-Tukey algorithm with sufficiently high resolution does not improve the performance as the SNR is increased (Fig. 3a, 3b). It is illustrated in Fig. 4a that the performance of 2-D arrays is better than that of 3-D arrays with the same antenna positions on x-y plane. Furthermore, it is also shown in Fig. 4a that the regular arrays have the better performance than irregular arrays. Fig. 4b and Fig. 5a illustrate the high resolution of MUSIC in comparing with MVSE and BT for 2-D and 3-D arrays, respectively. Finally, in considering resolution of MUSIC (Fig. 5b), it is shown that there are slight differences in performance between regular and irregular arrays as well as between 2-D and 3-D arrays.

References

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